# Dimensional Characterization and Effective Area Estimation of the 35 mm Piston/Cylinder Assembly -PG0806

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**Abstract:**
Piston/cylinder assemblies are widely used in the calibration of pressure measuring equipment whose principle of operation is to generate pressure by positioning non-standard weights which generate radial forces when compressing the fluid. The dimensional characterization of the piston/cylinder assemblies is fundamental to the realizations the pressure measurement made with piston/cylinder assemblies. There diameter, straightness and roundness measurements were made for the PG 0806 of the pressure laboratory of the INM - Colombia, both for the cylinder and the piston itself. To do this measurement the Brown & Sharpe coordinate measuring machine (CMM) of the dimensional laboratory of the INM was used. With the measuring method described, we were capable to measure the effective area with a relative uncertainty about $9.7\text{x}10^{-5}$.

## 1. Introducción

The piston/cylinder assemblies are used to generate pressures with high accuracy, they composed by a hollow cylinder and a piston as seen in Figure 1. The function principle of the piston/cylinder assemblies is simple: the piston inside the cylinder both oriented vertically, is pressurized with a certain fluid moving the piston upward the cylinder; then by adding known weights on the piston/cylinder assembly, the equilibrium point is reached and with the piston rotating the pressure measure is realized.

The pressure can be determined by knowing the combined mass of the weights, the local acceleration of free fall and the effective area of the piston/cylinder assembly. Often the effective areas are determined through a calibration to another piston gauge or from dimensional measurements, we are going to develop the second method.

The procedure to determine the effective area of piston-cylinder assemblies from the dimensional properties of the pistons and cylinders is very well documented (Ruiz [3]), for the straightness, roundness, and diameters, and the contributions to the uncertainty of them should be determined.

In the Figure 1, we can see the three reference levels for the measurement of the roundness and diameter both for the piston and the cylinder, in order to obtain the dimensional measurements of the effective area [3].

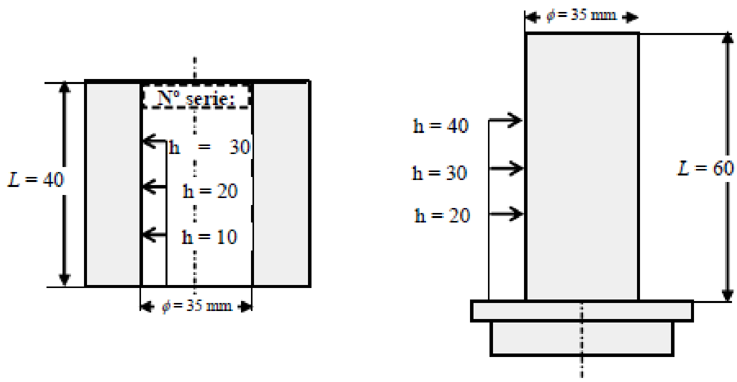


Figure 1. Measurement scheme [4].

The measurement of the dimensional properties of diameter and roundness of the PG 0801 were made with the coordinate measuring machine (CMM) Global Performance size 07.10.07 [1]. The selected stylus tip was a ceramic hemispherical 18 (mm) diameter (see Figure 2) [2].

Figure 2. Hemispherical ceramic probe (stylus tip) *[2]*.

## 2. Materials/methods

### 2.1 Measurement

In order to measure the diameter and roundness both for piston and cylinder the CMM Brown & Sharpe model 07.10.07 Performance was chosen.

### 2.1.1 Alignment and measurements of the cylinder.

The cylinder is alignment by hand using the top plane of the cylinder as the working plane and the direction parallel to its main axis as the Z-axis (see Figure 3). To achieve the origin of the system we used a distribution of four points equally spaced around the circle [3] (see Figure 4). After that the automatic align was made. To define the Z-axis a cylinder was generated as shown on Figure 5. After that the align was made (see Figure 6), and starting from the origin and the upper surface of the cylinder the diameter was measured from top to button at the heights: [1, 4.25, 11.25, 14.75, y 18.25] (mm) (see Figure 7).

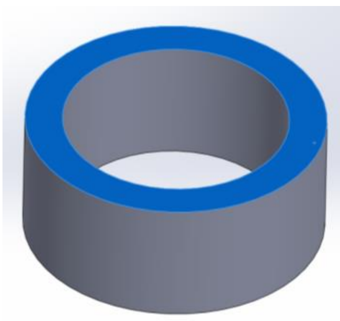

Figure 3. Upper Plane

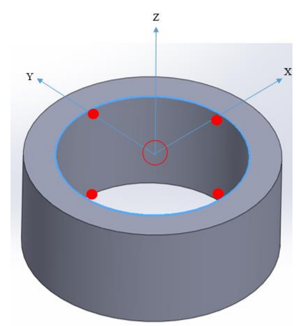


Figure 4. Origin point with the circumference made with four points.

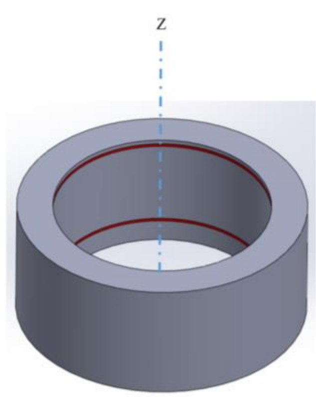


Figure 5. Z-axis

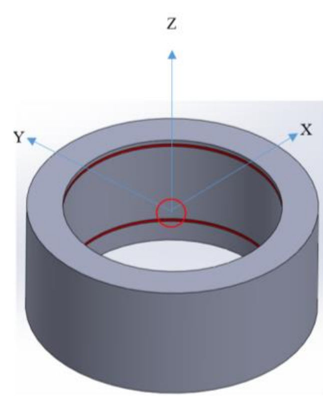


Figure 6. Alignment

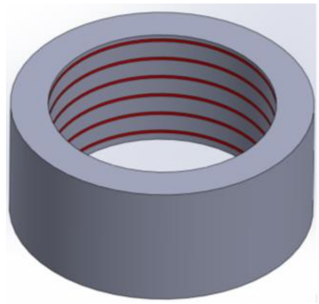

Figure 7. Measurement Heights

Each circumference was measured three times with: 36, 72, 84, 96, 108 and 120 contacts each.

### 2.1.2 Alignment and measurements of the piston.

The piston was aligned using the bottom face of the piston as working plane, placing it face-up (see Figure 8), the Z-axis is defined as perpendicular to this plane. In the same way that we did with the cylinder, a four point circle was made and the origin was set just as in the Figure 9 [3]. After that, the align was made automatically, once again the upper surface of the piston was used to generate the working plane. The generated cylinder that defines the Z-axis can be seen in Figure 10. After the align the system is left as can be seen in Figure 11.

Figure 8. Working plane and definition of Z-axis

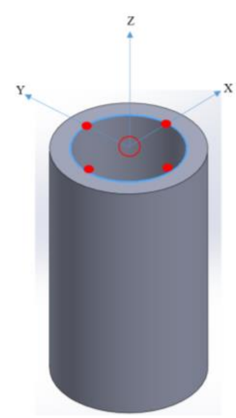


Figure 9. Origin referenced to a four points circumference.


Figure 10. Z-axis

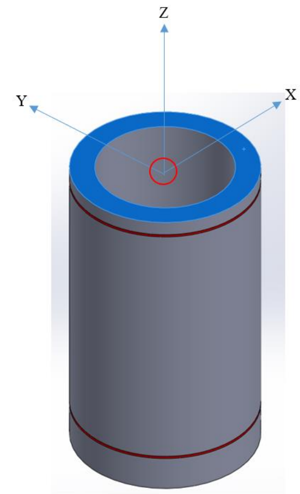


Figure 11. Automatic align

The diameter of the cylinder was measured from top to bottom at this heights: [3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54, 57] (mm) [3], as can be seen on Figure 12. With this data the roundness can be determined. Each circumference was measured three times with 120 points. The environmental conditions of the measurements were: ambient temperature between 19 °C - 21 °C and relative air humidity between 40% to 60 %.

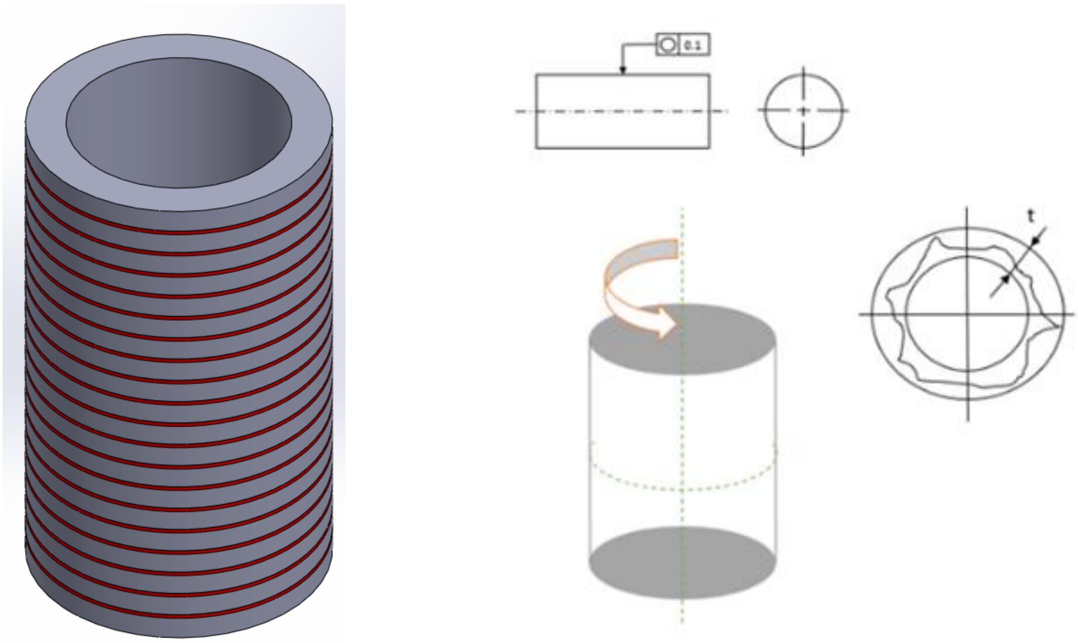


Figure 12. Heights for the roundness measure.

2.2. Effective area by geometric measurement.

The effective area of the piston cylinder assembly is assumed to be the average between piston and cylinder:

$$A_0 = \frac{(A_{0c} + A_{0p})}{2} \qquad \text{Equation 1}$$

Where $A_{0p}$ is the mean piston cross sectional area and $A_{0c}$ is the mean cylinder cross sectional area. Both are calculated by using the equation:

$$A_{0c,p} = \pi \frac{d^2{}_{c,p}}{4} \qquad \text{Equation 2}$$

Where $d_{c,p}$ is the inner diameter of the cylinder or the external diameter of the piston. For the uncertainty budget we use the equation 3 [4]:

$$\frac{u(A_0)}{A_0} = \left[\left(\frac{u_d(A_0)}{A_0}\right)^2 + \left(\frac{u_{nr}(A_0)}{A_0}\right)^2\right]^{\frac{1}{2}} \qquad \text{Equation 3}$$

Where $u_d(A_0)$ is calculated by using the equation 4:

$$\frac{u_d(A_0)}{A_0} = \frac{\sqrt{u_p^2 + u_c^2}}{2R_0} \qquad \text{Equation 4}$$

Where $u_p$ and $u_c$ are the uncertainties of the piston and cylinder measurement, respectively. Finally, $u_{nr}(A_0)$ is the uncertainty due to the lack of roundness and is obtained from equation 5:

$$\frac{u_{nr}(A_0)}{A_0} = \frac{\sqrt{u_{nr\,p}^2 + u_{nr\,c}^2}}{2R_0} \qquad \text{Equation 5}$$

## 3. Result

### 3.1 Measurements

#### *3.1.1.* Cylinder Measurements.

The mean values of the cylinder diameter at different heights can be seen in Table 1:

Table 1. Mean diameters values of the cylinder.

| Cylinder | | |
|---|---|---|
| Contacts Number | Mean Diameter (mm) | Roundness (mm) |
| 36 | 35.334 4 | 0.001 0 |
| 72 | 35.334 5 | 0.001 2 |
| 84 | 35.334 4 | 0.001 3 |
| 96 | 35.334 4 | 0.002 1 |
| 108 | 35.334 7 | 0.001 3 |
| 120 | 35.334 2 | 0.001 4 |

Whereas contacts number has an impact about the spend time to get a measure, the uncertainty does not change significantly when the contacts number are 36 or 120. The average diameter of the cylinder with 36 contact is 35.334 4 ± 0.003 4 (mm). There are twelve sources of uncertainty that were considered for that measurement, which are in the next table:

Table 2. Uncertainty budget.

| Sources of uncertainty | Typical uncertainty | Distribution type | Sensitivity coefficient | Contribution | $v$ |
|---|---|---|---|---|---|
| Uncertainty due to Maximum Permissible Error the coordinate measuring machine | $u(mach) = \frac{MPE_e}{\sqrt{3}}$ | B | 1 | 1.16 x $10^{-3}$ (mm) | 200 |
| Machine Calibration uncertainty | $u(cal) = \frac{U_3}{k}$ | B | 1 | 0.39 x $10^{-3}$ (mm) | 200 |
| Stylus tip qualification uncertainty | $u_{sph} = \sqrt{\frac{e_f^2}{\sqrt{3}} + \frac{U_{quasph}^2}{2}}$ | B | 1 | 0.22 x $10^{-3}$ (mm) | 200 |
| Uncertainty about Stylus tip settings | $u_{conf} = \frac{desv_{max}}{\sqrt{3}}$ | B | 1 | 0.75 x $10^{-3}$ (mm) | 50 |
| Uncertainty due to Non-orthogonality | $u_{noort} = \frac{r(1 - cos\alpha)}{\sqrt{3}}$ | B | 1 | 0.79 x $10^{-3}$ (mm) | 50 |

| | | | | | |
|---|---|---|---|---|---|
| Uncertainty by Resolution | $u_{res} = \frac{res}{2\sqrt{3}}$ | B | 1 | 0.03 x $10^{-3}$ (mm) | 200 |
| Uncertainty by Repeatability | $u_{rep} = \frac{Desv.std}{\sqrt{n}}$ | A | 1 | 0.11 x $10^{-3}$ (mm) | 11 |
| Uncertainty by Probe System Mount | $u(e_{psm}) = \frac{Max_{psm} - Min_{psm}}{2\sqrt{3}}$ | B | 1 | 0.75 x $10^{-3}$ (mm) | 50 |
| Uncertainty of the coefficient of thermal expansion (CTE) of the piece | $u(\alpha_p) = \frac{1 \times 10^{-6}}{\sqrt{3}\ °\mathrm{C}}$ | B | $-li.\theta_p$ | −0.01 x $10^{-3}$ (mm) | 50 |
| Uncertainty due to temperature over 20 °C | $u(\theta_p) = \sqrt{u(pter)^2 + u(rest)^2 + u(tl)}$ | B | $-li.\alpha_p$ | −0.23 x $10^{-3}$ (mm) | 50 |
| Uncertainty due to coefficient of thermal expansion with respect to 20 °C of the coordinate measuring machine | $u(\alpha_s) = \frac{0.5\ x\ 10^{-6}}{\sqrt{3}°C}$ | B | $li.\theta_e$ | 7.1 $\times$ $10^{-6}$(mm) | 50 |
| Uncertainty due to temperature differential over 20 °C of the coordinate measuring machine | $u(\theta_s) = \sqrt{u(pter)^2 + u(rest)^2 + u(tl)}$ | B | $li.\alpha_e$ | 0.23 x $10^{-3}$ (mm) | 50 |
| Combined uncertainty $u_c(x)$ | | | | 1.7 x $10^{-3}$ (mm) | |
| $k$ | | | | 2 | |
| $U$ | | | | 3.4 x $10^{-3}$ (mm) | |

Where $MPE_e$ is the maximum permissible error of manufacturer's specifications, $U_3 = 0.7 + \frac{L}{500}$ where $L$ is the length in mm of the coordinate measuring machine calibration certificate, $e_f$ is the master ball error, $U_{calesf}$ is the uncertainty of master ball, $desv_{max}$ is the deviation of the stylus tip when it is qualify, $\alpha$ is the angle between the stylus tip and the workpiece surface that has a value of 1°, $Desv.std$ is the deviation of the number of measurements (12) from the four levels with three measurements for each one, $Max_{psm}$ has a value of 1.4 and $Min_{psm}$ has a value of 1.2, $u(\theta_p)$ is the uncertainty by the reading of the temperature sensors of the piece, $pter$ is the uncertainty of the machine´s thermometers, $rest$ is the resolution of thermometers, $tl$ is the temperature variation of the laboratory which is 1 °C.

### *3.1.2.* Piston Measurements.

The mean values of the piston diameter at different heights with 120 contacts are recorded in the Table 3.

Table 3. Mean Diameter of the piston at different heights with 120 contacts.

| Piston - External Diameter | | | | |
|---|---|---|---|---|
| Diameter (d) (mm) | Roundness (mm) | Position | | |
| | | x | y | z (b) |
| 35.332 5 | 0.001 3 | 0.000 0 | 0.001 0 | -3.000 0 |
| 35.332 6 | 0.003 8 | 0.000 6 | 0.001 4 | -6.000 0 |
| 35.332 6 | 0.001 8 | 0.000 5 | 0.001 0 | -9.000 0 |
| 35.332 7 | 0.002 0 | 0.000 4 | 0.000 5 | -12.000 0 |
| 35.332 9 | 0.001 8 | 0.000 2 | 0.000 3 | -15.000 0 |
| 35.332 9 | 0.002 3 | -0.000 6 | -0.000 2 | -18.000 0 |
| 35.332 9 | 0.001 7 | -0.001 4 | -0.000 6 | -21.000 0 |
| 35.333 0 | 0.001 5 | -0.001 2 | -0.001 0 | -24.000 0 |
| 35.333 1 | 0.001 6 | -0.000 5 | -0.001 1 | -27.000 0 |
| 35.333 0 | 0.002 2 | -0.001 5 | -0.001 6 | -30.000 0 |
| 35.333 0 | 0.002 3 | -0.001 9 | -0.002 1 | -33.000 0 |
| 35.333 0 | 0.001 7 | -0.001 6 | -0.002 5 | -36.000 0 |
| 35.332 9 | 0.001 4 | -0.002 5 | -0.002 8 | -39.000 0 |
| 35.332 9 | 0.003 4 | -0.001 9 | -0.003 4 | -42.000 0 |
| 35.332 9 | 0.002 2 | -0.002 2 | -0.003 7 | -45.000 0 |
| 35.332 7 | 0.002 3 | -0.002 7 | -0.004 6 | -48.000 0 |
| 35.332 8 | 0.002 3 | 0.0032 0 | -0.004 4 | -51.000 0 |
| 35.332 5 | 0.002 1 | -0.002 9 | -0.004 7 | -54.000 0 |
| 35.332 3 | 0.003 7 | -0.003 2 | -0.005 0 | -57.000 0 |

The mean diameter is 35.332 8 (mm).

## 4. Discusión

### 4.1 Effective Area

The effective area of the piston cylinder assembly is calculated using Equation 2 with the mean values of the diameters of the cylinder and the piston taken from the Table 1 and Table 3 with 84 and 120 number of contacts each, the area values are in the Table 4:

Table 4. Mean diameter and area for Piston and cylinder

| Piston | | Cylinder | |
|---|---|---|---|
| d (mm) | Sd (mm) | d (mm) | Sd (mm) |
| 35.332 8 | 0.000 2 | 35.334 4 | 0.000 4 |
| Area (mm$^2$) | Sd (mm) | Area (mm$^2$) | Sd (mm) |
| 980.496 4 | 0.011 7 | 980.585 2 | 0.000 4 |

Starting from the Equation 1 and using the values of the area of the piston and cylinder, the effective area was calculated by 980.540 7 (mm$^2$). The known cross-floating effective area (PTB 2007) is 980.533 (mm$^2$), then there is a difference of 0.008 (mm$^2$).

### 4.2 Effective Area Uncertainty.

From equation 3 the uncertainty of the effective area was calculated, see Table 5.

Table 5. Variables for the determining the uncertainty of the effective area.

| Quantity | Symbol | Value |
|---|---|---|
| Coverage factor | $k$ | 2 |
| Piston Diameter Combined Uncertainty | $u_p$ | 1.7 x $10^{-3}$ (mm) |
| Cylinder Inner Diameter Combined Uncertainty | $u_c$ | 1.7 x $10^{-3}$ (mm) |
| Cylinder Roundness Combined Uncertainty | $u_{pr}$ | 1.65 x $10^{-3}$ (mm) |
| Piston Roundness Combined Uncertainty | $u_{cr}$ | 1.7 x $10^{-3}$ (mm) |

| | | |
|---|---|---|
| Diameter Uncertainty | $u_d$ | 6.87E-05 x $10^{-3}$ (mm) |
| No Roundness Uncertainty | $u_{nr}$ | 6.77E-05 x $10^{-3}$ (mm) |

Then the effective area relative uncertainty is 9.64 x $10^{-05}$.

## 5. Conclusión

By developing the method to realize the dimensional characterization to calculate the effective area of the PG 0806 of the INM Pressure Laboratory a new branch of dimensional metrology has been developed.

With the measure of the roundness and the diameter of the piston cylinder system we were capable to determine the effective area and its related uncertainty. The effective area of the PG0608 obtained by this method 980.5407 ($mm^2$) with a relative uncertainty of 9.64 x $10^{-5}$.

A method to measure the effective area for the PG 0806 was successfully developed, unfortunately the uncertainty of the measure is too high to allow a geometric realization of the Pascal. Nevertheless, with data from higher precision machine it would be possible to reach the desired uncertainty, because with the coordinate measurement machine the accuracy is 2 x $10^{-3}$ (mm).

## 6. References

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